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4th Symposium on "Hypercompositional Algebra-new Developments and Applications"

Probabilistic polygroup theory

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Probabilistic group theory is concerned with the probability of a group's elements or a group's subgroups satisfying certain conditions. On the other hand, a polygroup is a generalization of a group and it is a special case of a hypergroup. This presentation studies probabilistic polygroup theory as an extension of probabilistic group theory. In this regard, we extend the concepts of the (relative) subgroup commutativity degree of a finite group to the subpolygroup commutativity degree of a finite polygroup and to relative commutativity degree of a subpolygroup K of a polygroup P, respectively. Furthermore, we present some of their properties. We then consider two special polygroups; the polygroup associated to a group and the extension polygroup, and obtain explicit formulas for their commutativity degrees.

Quasi filters and *(n,m)*-quasi filters on ordered semirings: first results

FAKHRY ASAD AGUSFRIANTO
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Ordered structures is the structure that involve partially ordered (*po*) relation in its mathematical system. The notion of (*po*) can applied in algebraic and hyperalgebraic structures such as semigroups, semirings, semihypergroups, and semihyperrings. Furthermore, in po-semigroup, there is a notion called filters. This notion first investigate by Kehayopulu in 1987. Next, Al-Tahan introduced a new types of filters in ordered semigroups in 2022, that is quasi filters and *(m,n)*-quasi filters. Inspired by previous research about quasi filters and *(m,n)* quasi filters in or-

dered semigroups, this paper aim to introduced the notion of quasi filters and *(m,n)*-quasi filters in ordered semirings. Furthermore, some properties related to quasi filters and *(m,n)*-quasi filters in ordered semirings are also investigate.

The class of Krasner hyperfields is not elementary

PIOTR BŁASZKIEWICZ
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During my talk I will show that the class of Krasner hyperfields (i.e. hyperfields obtained from fields by quotient construction introduced by Krasner) is not elementary. Answering this basic model theoretical question leads to some interesting general algebraic results, about the rational rank of quotients of multiplicative groups in field extensions. I will present how these two questions are connected.

The talk is based on the preprint "The class of Krasner Hyperfields is not elementary" by Piotr Kowalski and myself.

Categorical approach of normal injective and torsionable hypermodules

HASHEM BORDBAR
University of Nova Gorica, Slovenia

Inspired by the characterization of injective objects in category theory, our main purpose in this research is to investigate the relationships of divisible and injective hypermodules with torsion elements and torsionable hypermodules with a category approach. These days, it is extremely tempt- ing to do the research in an Abelian category, so we discuss these topics in an Abelian category. For a Krasner R-hypermodule M, we extend the definition of a zero-divisor element of R to a zero-divisor element of R over M, and then the definition of divisible R-hypermodule is introduced. Be- sides, the torsion element and torsionable hypermodule are introduced and we show that every torsionable R-hypermodule M is a normal injective, where R is a commutative hyperring.

Hypercompositional structures and relational monoids

NICOLÒ CANGIOTTI and ALESSANDRO LINZI
Polytechnic University of Milan, Italy
University of Nova Gorica, Slovenia

The familiar category **Set** of sets and functions has a 'half-sibling': the category **Rel** formed by sets and binary relations. In this talk, we will focus on monoids in **Rel**, which are defined by analogy with monoids in **Set** (i.e., standard monoids).

We will explain how associative hypercompositional structures with a scalar identity, as well as certain structures with a partial operation may both be interpreted as such 'relational monoids'.

The proposed study is mainly motivated by recent research work aiming to develop a less tedious formalism for Quantum Theories, availing of the diagrammatic nature of the language of categories. In fact, within this ambitious program, categories which are fundamentally more similar to **Rel** than to **Set** find many applications. During the discussion, we will sketch some lines of investigation where we believe hypercompositional algebra should play a role.

An Overview of HX-Groups

IRINA CRISTEA
University of Nova Gorica, Slovenia

HX-groups were introduced around 1985 by a group of Chinese mathematicians, initially under the name of "hypergroups", being considered as "an upgrade structure of groups". Indeed, they are groups defined on the collection of the non-empty subsets of a given group, having strong similarities with a particular type of hypergroups. The aim of this presentation is to clarify the definition and offer an overview of the main properties of HX-groups (regularity, uniformity, essentiality), as well as to suggest some open problems on this topic.

Homological Algebra in Abelian Categories

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Institute for Advanced Studies and Basic Sciences, Iran

Homological algebra is a branch of mathematics that studies chain complexes, derived functors and their applications. In the classical homological algebra, which was systematically introduced by Cartan and Eilenberg, all the focuses are on the category of R-modules and additive functors from the category of R-modules to the category of S-modules, where R and S denote two rings. However, the applications of homological algebra as a very useful tool, are much wider than the category of modules and functors between them.

In this talk, I will review some basic definitions and then generalize the concepts of classical homological algebra from the category of modules to Abelian categories just as Cartan and Eisenburg’s work. Roughly speaking, an Abelian category is a category satisfying just enough axioms so the snake lemma holds. The proof of snake lemma is mostly based on universal properties of these categories.

Isomorphism classes of join spaces associated with chains

VIOLETA LEOREANU-FOTEA
University Alexandru Ioan Cuza of Iaşi, Romania

In this presentation we analyze the isomorphism problem of two join spaces associated with a chain.
Both the finite and the infinite case are analyzed.

Valued Fuzzy Superhypergraphs and Applications in Decision Making

MOHAMMAD HAMIDI and FLORENTIN SMARANDACHE
University of Payame Noor, Iran
University of New Mexico, United States of America

In this presentation we will discuss about the defects in fuzzy (hyper) graphs (as complex (hyper) networks) and extends the fuzzy (hyper) graphs to fuzzy (quasi) superhypergraphs as a new concept. We have modeled the fuzzy superhypergraphs as complex superhypernetworks in order to make a relation between labeled objects in the form of details and generalities. Indeed, the structure of fuzzy (quasi) superhypergraphs collects groups of labeled objects and analyzes them in the form of the part to part of objects, the part of objects to the whole group of objects, and the whole to the whole group of objects at the same time. We have investigated the properties of fuzzy (quasi) superhypergraphs based on any positive real number as valued fuzzy (quasi) superhypergraphs, considering the complement of valued fuzzy (quasi) superhypergraphs, the notation of isomorphism of valued fuzzy (quasi) superhypergraphs based on the permutations, and we have presented the isomorphic conditions of (self-complemented) valued fuzzy (quasi) superhypergraphs.

The concept of impact membership value of fuzzy (quasi) superhypergraphs is introduced in this study and it is applied in designing the real problem in the real world. Finally, we have presented the problem of business superhypernetworks as an application of fuzzy valued quasi superhypergraphs in the real world.

Hypergroups arising from the standard particle physics

MORTEZA JAFARPOUR
Vali-e-Asr University, Iran

In this research we introduce the class of interaction hypergroups in which the combination of two elements in a hypergroup arising from in-

teractions between Leptons in the Standard Model of elementary particles. We show that the set of all Leptons with hyperstructure based on conservation laws forms an interaction hypergroup.

Graph Hyperstructures

ANTONIOS KALAMPAKAS
American University of the Middle East, Kuwait

We explore hyperstructures on graphs, focusing on the application of hyperoperations to analyze relationships within directed graphs. We introduce three key types of hyperoperations: the Path Hyperoperation, the Simple Path Hyperoperation, and the Ancestry Hyperoperation.

The Path Hyperoperation identifies all vertices lying on paths between pairs of vertices, providing a comprehensive view of connectivity within the graph. The Simple Path Hyperoperation refines this by considering only simple paths, where each vertex is visited at most once, offering insights into non-redundant routes within the graph. The Ancestry Hyperoperation determines common ancestors based on paths leading to vertices, highlighting shared lineage and influences within the network.

Through detailed examples and the construction of multiplication tables for each hyperoperation, we demonstrate how these operations can be used gain insight on graph structures which can lead to applications in areas such as network analysis, computational biology, and social network analysis. We also also outline the properties of these hyperoperations and their implications for understanding graph structure and dynamics.

We conclude by discussing potential future work, including extending these concepts to weighted and dynamic graphs and developing efficient algorithms for large-scale graph analysis in order to provide a foundation for leveraging hyperstructures and gain deeper insights into the intricate relationships and connectivity within graphs.

A note on hypermodules and hyperrings

ENGIN KAYNAR, BURCU NİŞANCI TÜRKMEN , ERGUL TÜRKMEN
Amasya University, Turkey

The main purpose of this talk is to study the concept of the hyperring $(\mathbb{N},\oplus,.)$, where $m\oplus n = \{m+n, k|\ \min[m,n\}+k=\max\{m,n\}\}$ for all m, n and the operation . is the usual multiplication in $\mathbb{N}$. In particular, we prove that this hyperring $(\mathbb{N},\oplus,.)$ is isomorphic to Krasner's quotient hyperring. Moreover, we construct the hyperstructure $(\mathbb{N}_{\mathbf{m}},\oplus_{\mathbf{m}},.)$,which is a class of examples of hypermodules and hyperrings. In addition, we show

that a hyperring R (not necessarily unitary) can be embedded the hyperring R with identity.

Remarks and comments on the axioms of hypercompositional structures

CHRISTOS MASSOUROS and GERASIMOS MASSOUROS
National and Kapodistrian University of Athens, Greece
Hellenic Open University, Greece

It is imperative to apply thorough and meticulous consideration to the independence of the axioms used to define mathematical entities. For instance, the independence of the parallelism postulate from the rest axioms of Euclidean geometry occupied Mathematics for more than a millennium and gave rise to the study of two non-Euclidean Geometries. In this presentation, we are going to show that we can restrict the number of axioms we used to define numerous hypercompositional structures. In addition to everything else, this facilitates the use of computational techniques to construct and enumerate these hypercompositional structures.

Actions of Automata Created by Chains of Differential Neurons

JAN CHVALINA, MICHAL NOVAK and BEDŘICH SMETANA
Brno University of Technology, Czech Republic

When searching for new possibilities of modelling and descriptions of time processes, one may turn the attention to actions of hypercompositional structures on phase spaces of various operators. In this respect, results regarding systems of artificial neurons, in particular of the differential type, seem to be promising. In our presentation we give an overview of our recent introductory results in our study of artificial neurons from the point of view of hypercompositional structures.

Dependence relations, directed graphs and degree of influence

ANTON NUCULOVIĆ
University of Montenegro, Montenegro

The aim of this presentation is to offer an overview on the new theory of dependence relations. They can model, from an algebraic point of view, interrelations between different sets of elements. After presenting the definition and main properties of the dependence relations, we will recall how to associate a directed graph to a dependence relation. Then a

partial hyperoperation is associated with the directed graph, for which a pre-degree and a degree of influence are calculated. These two numerical parameters measure the strength of the influence of one element on another within a given set of elements (variables).

Hyperalgebras and Abstract Theories of Quadratic Forms from a Functorial Point of View

KAIQUE ROBERTO
University of São Paulo, Brazil

We provide equivalences and dual equivalences between categories of abstract theories of quadratic forms and subcategories of hyperfields/multirings, that brought new perspectives and methods to the abstract theories of quadratic forms.

Study of Cayley digraphs on polygroups

ALI SANJABI
Vali-e-Asr University, Iran

Cayley digraphs are directed graphs associated to a polygroup and a set of finite generators for that polygroup. In this paper examine multiple examples of Cayley digraphs through polygroup theory, graph theory and applications. We are investigating some properties of the Cayley digraph on a polygroup such as connectivity and existance of cycles for each member of the graph. Also we identify Cayley digraphs on polygroups derived from conjugate classes of dihedral groups.

Description of HX-groups using the C++ programming language

ANDROMEDA SONEA-PATRASCU
Iaşi University of Life Science, Romania

The central concept of our work is to elucidate the HX-groups with the dihedral group D_n as support, leveraging the power of the C++ programming language, providing a robust foundation, and enriching our research.

The program, built in Microsoft Visual Studio 2022, proved the commutativity of two HX-groups. From an algebraic perspective, composing two HX-groups with the dihedral group as support is complex and time-intensive. However, the use of the C++ programming language effectively reduced this complexity.

A new vulnerability in Elliptic Curve Cryptography

ENRICO TALOTTI
University of Nova Gorica, Slovenia

The strength of Elliptic Curve Cryptography (ECC) relies on the intractability of the Discrete Logarithm Problem within a prime-order group of rational points of an elliptic curve. Many algorithms have been proposed to solve this problem and break the elliptic curve cryptosystem, yet all require exponential running time with a traditional machine. In this work we investigate a group action defined on the group of rational points of an elliptic curve. This will reveal an improvement of the computation of the discrete logarithm under certain conditions, uncovering a subtle type of "weak" private keys that inherently exist in every cryptosystem based on the discrete logarithm problem, set in a group of prime order p such that $p-1$ has many small divisors.

We quantify weak key prevalence across standardized curves, highlighting a potential vulnerability due to numerous small divisors of auxiliary group orders. This study underscores the importance of careful curve selection and key generation to mitigate such vulnerabilities.

An algorithmic approach to the creation of hypercompositional structures

CHARALAMPOS TSITOURAS
National and Kapodistrian University of Athens, Greece

We present a Mathematica package (a symbolic manipulation tool) designed to process a set of hypergroupoids and identify those that qualify as hypergroups. This user-friendly program also categorizes hypergroups of a specific order into isomorphic classes and provides their cardinalities.

On τ-supplements in subcategory $(\_R\_S\hat{}) hmod$

BURCU NİŞANCI TÜRKMEN AND BİJAN DAVVAZ
Amasya University, Turkey
Yazd University, Iran

In this talk, we define radical of Krasner hypermodules in the subcategory $(\_R\_S\hat{}) hmod$, then we use short exact sequences in homological algebra for Krasner hypermodules. Then we study the concept of τ-supplements in module theory will be generalized to Krasner R-hypermodules by using short exact sequences and subcategory of $(\_R\_S\hat{}) hmod$.

The Linear Transformations which Unify the Newtonian Physics and Einsteinian Special Relativity and their Combination with FLRW metric for the Study of the Expansion of the Universe

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We have obtained two Linear Transformations:

i. Closed Isometric Complex Boost in Isotropic Complex Spacetime and

ii. Open Isometric Generalized Real Boost in Isotropic Real Spacetime.

Thus, we have a Generalized Special Relativity, which Unifies the Newtonian Physics and Einsteinian Relativity Theory. The combination of the Generalized Special Relativity with FLRW metric gives a flexible Cosmological Physics for the Study of our Universe which expands.