# 3rd Symposium on "Hypercompositional Algebra-new Developments and Applications"

## ABSTRACTS

### Comments on certain classes of hyperfields and hyperrings

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This presentation analyses the known classes of hyperfields and hyperrings and classifies some finite hyperfields as quotient or non-quotient ones.
Moreover, the problem of the isomorphism of certain hyperfields to the quotient hyperfields triggers open problems in the theory of fields that are listed in this presentation with their hitherto answers.

### The fuzzy degree of a genetic hypergroup

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In this lecture the fuzzy degree of a particular genetic hypergroup, associated with non-Mendelian inheritance is analyzed. It is considered a sequence of membership functions and of join spaces, obtained starting with a hypergroupoid. The fuzzy grade is the minumum natural number $i$ such that two consecutive associated join spaces, of the above mentioned sequence, $H_i$ and $H_{i+1}$ are isomorphic.

### An application of hyperstructures in the simple mendelian dominance

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Biologist and mathematician Gregor Mendel is considered the founder and father of genetics. He carried out research based on experimental hybridization in several species: peas, corn, beans, etc. The study of hyperstructures in this context is necessary because the results of the experiments in the case of the monohybrid cross in the pea plant, the dihybrid cross in the pea plant, the inheritance in the Four-o'clock plant, and so on describe hypergroups. This paper presents a combinatorial study for the distribution of phenotypes, starting with the cases of simple dominance for the dihybrid cross, trihybrid cross, and the 4-hybrid cross. And in final, we generalize these results. So, we determined a model for the type and number of phenotypes after several hybridizations.

## Some results on regular local hyperring and its relation with hyperdomains

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The study of regular local hyperrings has started by defining the regular parameter elements in a commutative local hyperring. It was proved that the dimension of a local hyperring $R$ with maximal hyperideal $M$ is equal to the height of $M$. Since $\frac{R}{M}$ is a hyperfield, the quotient hypermodule $\frac{M}{M^2}$ becomes a vectorial hyperspace, having the dimension greater than or equal to the dimension of the local hyperring $R$. In the case of equality, the hyperring is called regular. In this presentation, we continue to investigate more properties of regular local hyperrings, with the aim to prove that any regular local hyperring is a hyperdomain, i.e., a commutative hyperring without divisors of zero. More precisely, the study of the relationship existing between the dimensions of the vectorial hyperspaces $\frac{M}{M^2}$ and $\frac{\overline{M}}{\overline{M}^2}$ related to a local Krasner hyperring $R$ with maximal hyperideal $M$, where $\overline{M} = \frac{M}{<a>}$ and $a \in M \setminus M^2$ will be presented.

## Approaching General Relativity with methods and tools from the Special Relativity

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Contrarily to the prevailing perception, the gravitation of General Relativity (GR) can produce the corresponding gravitation of Special Relativity (SR) and vice-versa. The key-point of this correlation is the time dilation that originates from the metric of curved spacetime of GR. This implies the corresponding Lagrangian and the Gravitational Scalar Generalized Potential (GSGP) of GR. The reverse procedure is also feasible and leads to the corresponding Riemannian or non-Riemannian metric of GR. In fact, the SR gravity is associated to any kind of GR spacetime metric (including the non-Riemannian spacetimes with Finsler geometry) rather than the simple description of Einstein field equations of Riemannian GR. In this presentation, this is applied to attractive/repulsive black holes according to the Schwarzschild metric and Teleparallel gravity, wormholes with spherical symmetry and also to the Friedmann–Lemaître–Robertson–Walker metric for the explanation of the accelerating expansion of the Universe. Finally, it is shown that suitable gravitational fields (according to GR and SR) can accelerate particles to superluminal speeds.

## On the rev-transposition hypergroups

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In this presentation some first results from the rev-transposition hypergroups study are given.

## Some remarks on subpolygroup commutativity degree of finite polygroups

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The idea is to extend the concept of the subgroup commutativity degree of a finite group to subpolygroup commutativity degree of a finite polygroup $P$.

The latter measures the probability of two random subpolygroups of $P$ commuting. In this regard, some results concerning the new defined concept for groups and for polygroups will be mentioned. Moreover, the subpolygroup lattice of a special class of polygroups will be discussed. Finally, the explicit formulas for the subpolygroup commutativity degree of some special polygroups will be presented.

For future research, we will raise some open problems.

1. Find an explicit formula for the subpolygroup commutativity degree of other classes of finite polygroups.
2. For a finite polygroup $\langle P, \cdot, e, ^{-1} \rangle$, find the smallest positive real number $\alpha$ such that if $sd(P) \geq \alpha$ then $sd(P) = 1$.
3. Given finite polygroups $P_1, \ldots, P_n$, find a necessary and sufficient condition so that $sd(P_1 \times \ldots \times P_n) = sd(P_1) \ldots sd(P_n)$.
4. For a finite polygroup $\langle P, \cdot, e, ^{-1} \rangle$, find a necessary and sufficient condition so that $sd(P) = nd(P)$.
5. For a finite polygroup $\langle P, \cdot, e, ^{-1} \rangle$, find a relationship between its commutativity degree and its subpolygroup commutativity degree.

*Keywords and phrases:* polygroup, subgroup commutativity degree, subpolygroup lattice, subpolygroup commutativity degree.

In the presentation the results from the paper M. Al Tahan, S. Hoskova-Mayerova, B. Davvaz and A.C. Sonea *On subpolygroup commutativity degree of finite polygroups*, which is under review in AIMS, were presented.

## Parameters of hyperstructural model for autonomous car navigation

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Trajectories used in HD maps of autonomous vehicles can be well modeled by means of n-ary hyperoperations and hypergroups. For the applicability of such a model, the setting of the model parameters is important. We present this setup and propose an invertible trajectory model.

## The order relation in real Krasner quotient hyperfields

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In the theory of ordered fields with every positive cone $P$ one can associate a strict linear order defined as follows: $x < y$ if and only if $y - x$ is an element in $P$. In a real hyperfield with a positive cone $P$ we can define a relation in a similar way by saying that $x < y$ if and only if $y - x$ is a subset of $P$. However, in the case of real hyperfields this relation is a strict partial order.

Any strict partial order can be easily assigned to a directed graph. Thus, we can represent the structure of a real hyperfield $(F, P)$ by a directed graph. This was first done in the paper [1]. We develop this theory for the case of real quotient hyperfields $K/T$ and show what are the possibilities for a directed graph in this case. Moreover, we present the connection between a positive cone in a hyperfield $K/T$ and the corresponding positive cone in the field $K$ and show how the structure of the directed graph is related to the multiplicative subgroup T.

[1] Kędzierski, D.E.; Linzi, A.; Stojałowska, H. Characteristic, C-characteristic and Positive Cones in Hyperfields. Mathematics, 11, 779, 2023.

## Towards model theory of hyperfields

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Hyperfields play a special role in the model theory of the henselian valued fields of mixed characteristic. They are used, in the form of so called $RV$-sorts, to provide numerous results, such as relative quantifier elimination.

These particular objects are well known and well understood by the model theorists following the subject, however there is no model theory developed for the general theory of hyperfields.

The goal of this talk is to look at the theory of hyperfields in general. We shall investigate some natural model theoretical questions one needs to ask when approaching the new theory and propose suitable level of generality, that is less restrictive than just $RV$-sorts, but still promises some possible model theoretical results. We will focus on the question of the possibility of first order axiomatization of the class of Krasner factor hyperfields, and provide some partial results based on the work *The hyperring of adèle classes* of Alain Connes and Caterina Consani.

## Hypercompositional algebra and theoretical machines

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Theoretical machines (state machines, automata, multiautomata, etc.) are types of mathematical modeling tools that are commonly used to investigate how a system interacts with its environment. They are generally used when the system is considered or supposed to consist of discrete states that change in response to external distinct inputs. This presentation gives some results when the environment of a finite state machine is a two-element magma.

## Hypercompositional structures and quantum logic

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Since the beginning of its dissemination, quantum mechanics represents a suggestive mathematical challenge from many points of view. In particular, the differences between classical and quantum system have been formalized in terms of propositional logic and lattice theory.

The very first idea was introduced in 1936 by Birkhoff and von Neumann. This approach has led to interesting developments around the 70s. In this period, the most important con- tributions in this area are due to Jauch and Piron, who constructed a non-classical logical propositional system called quantum logic. We shall introduce some key concepts of this approach in view of a hypercompositional formalization based on well-known connections with lattice theory and projective geometry. This is the first part of a joint project with A. Linzi.

## Hypercompositional structures and quantum logic

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Recent work by Connes and Consani shows a fundamental connection between projective geometries and canonical hypergroups. On the other hand, various types of hypergroups arising from lattice structures have been studied by Nakano, Comer as well as Corsini and many others. We discuss the potential applications that these connections have in the description of quantum logic. This is the second part of a joint project with N. Cangiotti.

## An extension of the study on P1-P2 hypernear-rings

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In this note we aim to extend the basic results on a class of P1-P2 hypernear-rings. In this paper we show that under certain conditions the P1-P2 hypernear-ring is weak embeddable into multitransformation general hypernear-ring of its additive hypergroup. Moreover we show that there exists a subclass of this class of P1-P2 hyperrings that is weak-embeddable into a hyperring of multiendomorphisms associated with the additive hypergroup of the starting hyperring.

## Hyperrings of polynomials

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The main aim of this paper is the systematization of the previous results related to different classes of polynomial hyperrings and the construction of the new classes.

We will try to answer the open questions: if there exists a strong distributive subclass of the multiplicative hyperrings used by R. Processi Ciampi and R. Rota in their paper as a starting class over which it is possible to construct multiplicative hyperring of polynomials, and also the question of a contsruction of the Krasner hyperrings with identity, that satisfy the conditions from the paper [5], such that these examples are different from the classes constructed in [5] and [6].

Also, the aim is to construct new examples of polynomialy structured hyper- rings that satisfy certain conditions. We will study under which conditions we can apply Euclid's division algorithm in hyperrings of polynomials. In a case of the superring of polynomials, we will check whether the analogue of Hilbert's base theorem is valid, as well as whether certain generalizations of classical theorems related to polynomial rings are valid.

Some related references are listed below.

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## Hypergroups and arithmetic functions

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In this presentation several arithmetic functions related to hypergroups are recalled. These are the fuzzy grade, the commutativity degree, Euler's totient function and the degree of influence. We focus on the main properties, illustrated by several examples involving different types of hypergroups.